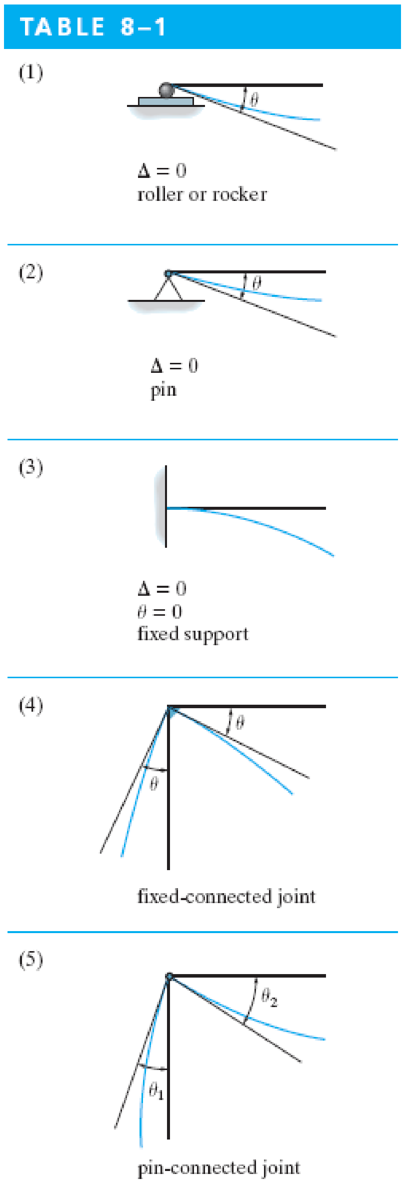
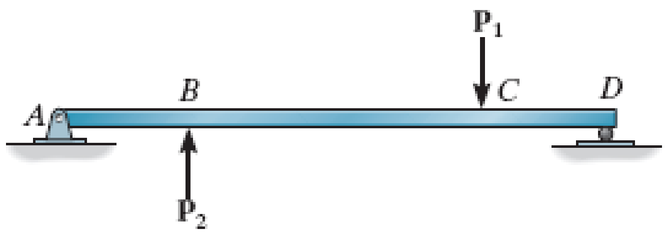
Deflection- Displacement & Rotation

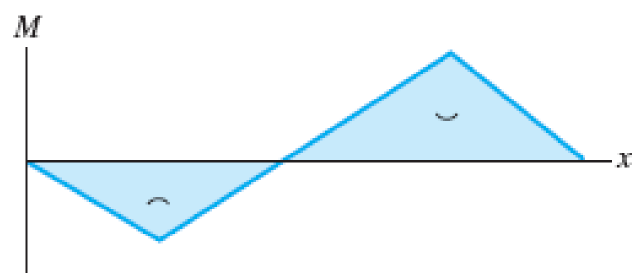
- Deflections maybe due to **loads, temperature, fabrication errors or settlement**
- **Linear elastic behavior:** linear stress-strain relationship and small deflection
- **Need to:**
  - Plot qualitative deflection shapes before/after the analysis
  - Calculate deflections at any location of a structure

Deflection Diagrams/Shapes

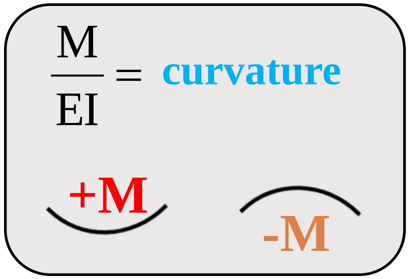


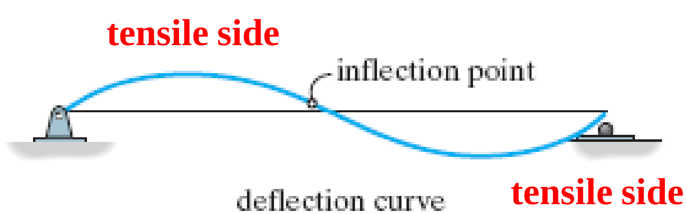


beam



moment diagram

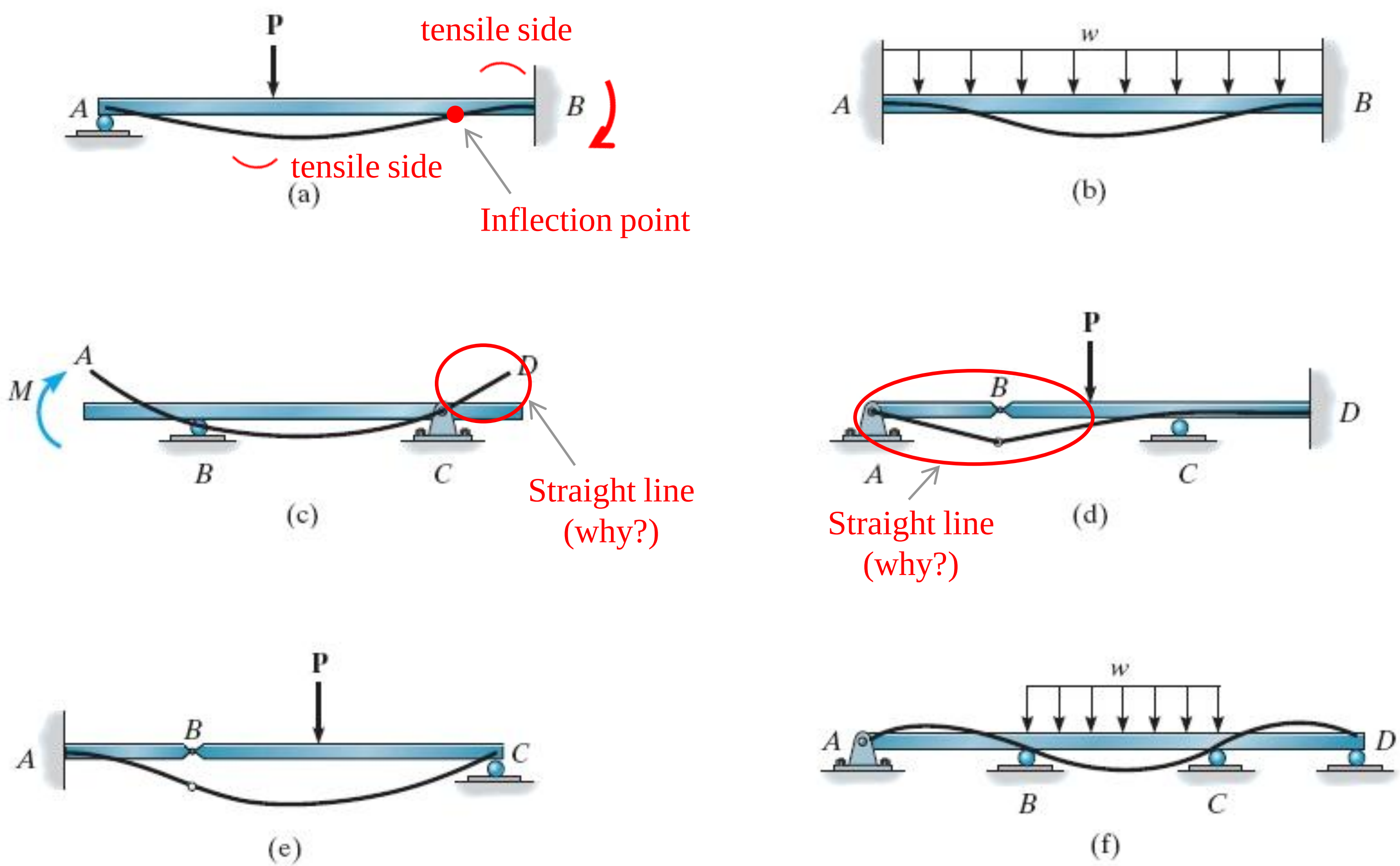




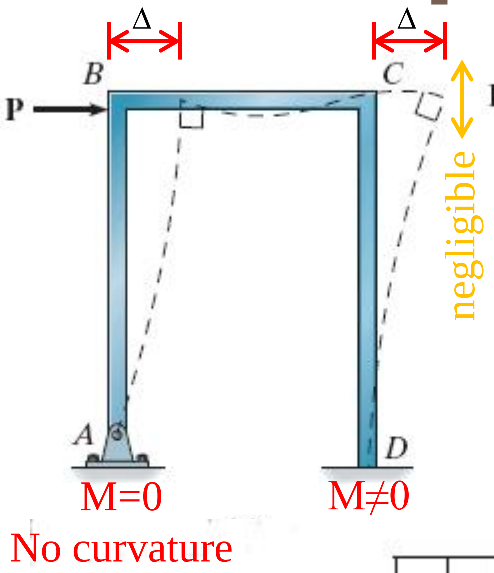
Example 8.1

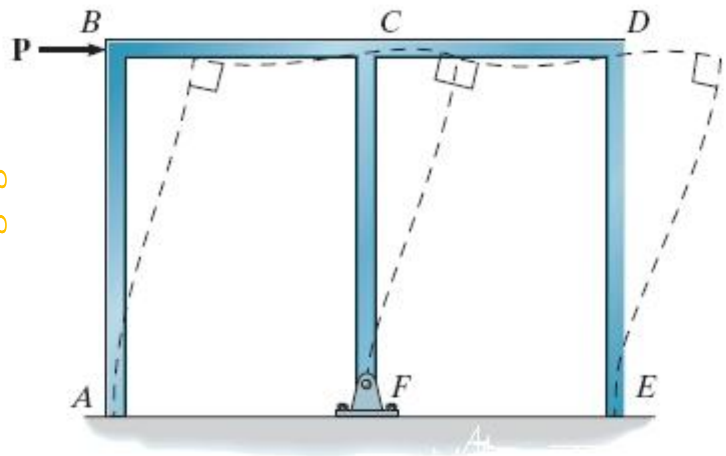
Draw the deflected shape of each of the beams.

Need to show 1st order (slope) and 2nd order (curvature) information

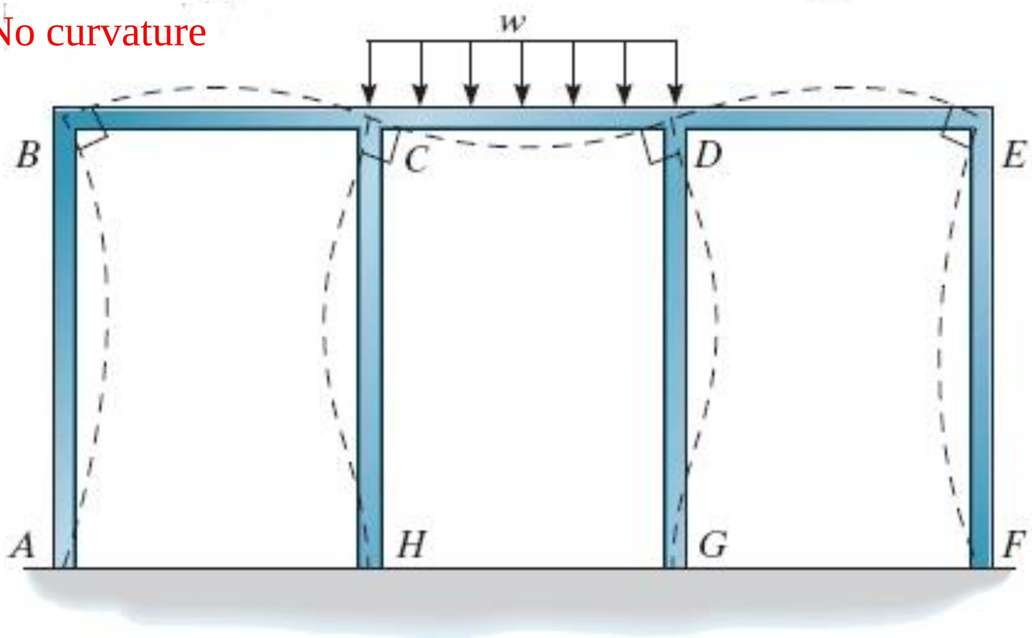


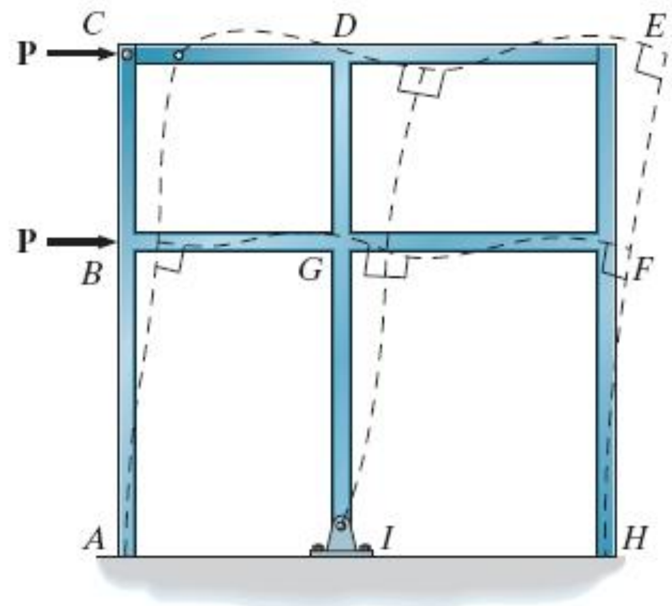
Example 8.2





All members are axially inextensible!





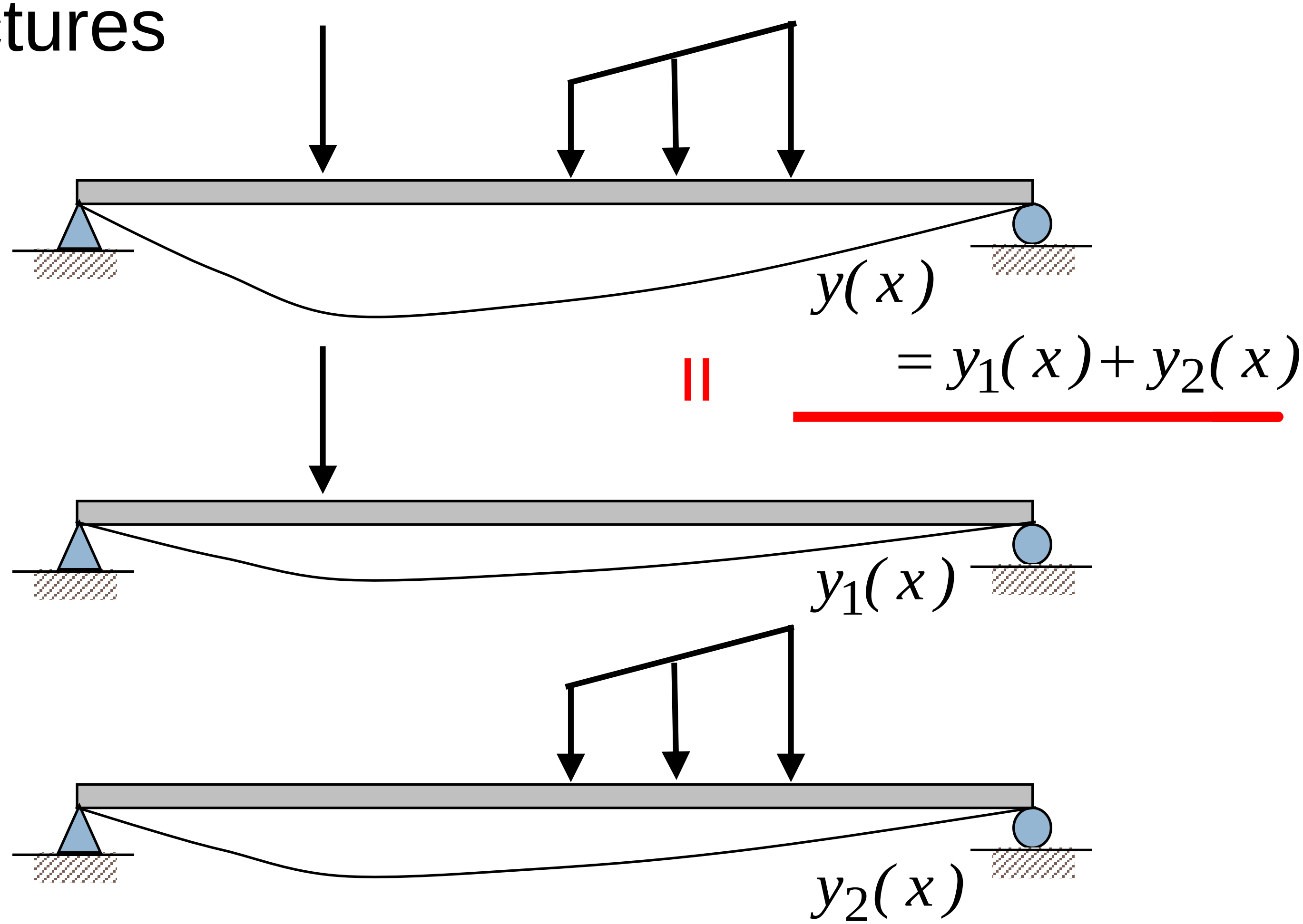
(d)

Calculation of Deflections

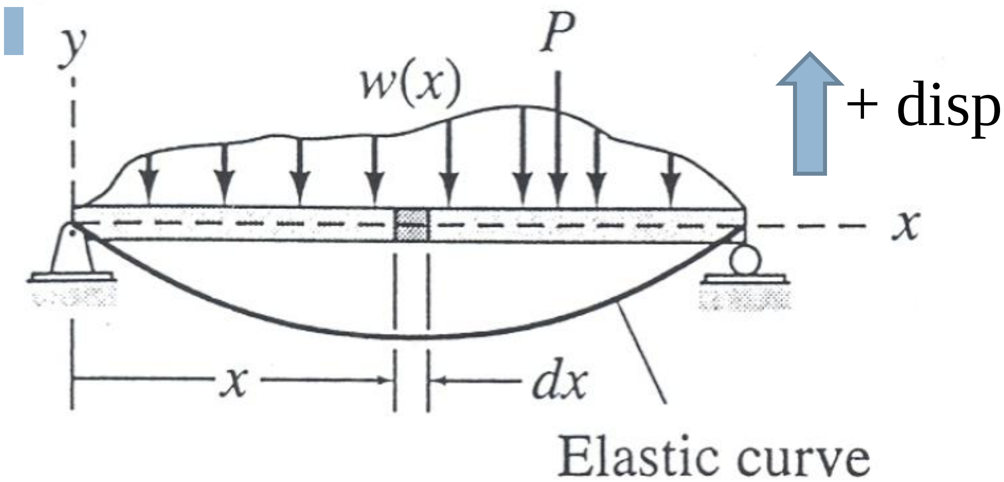
- Direct integration
- Moment-area method
- Conjugate-beam method
- Energy methods (in Chapter 9)

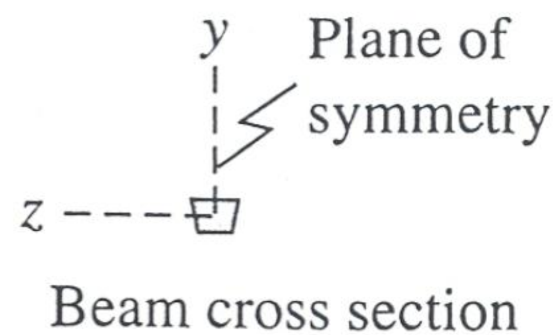
Calculation of Deflections

- Note: Superposition can be used for linear structures



Direct Integration



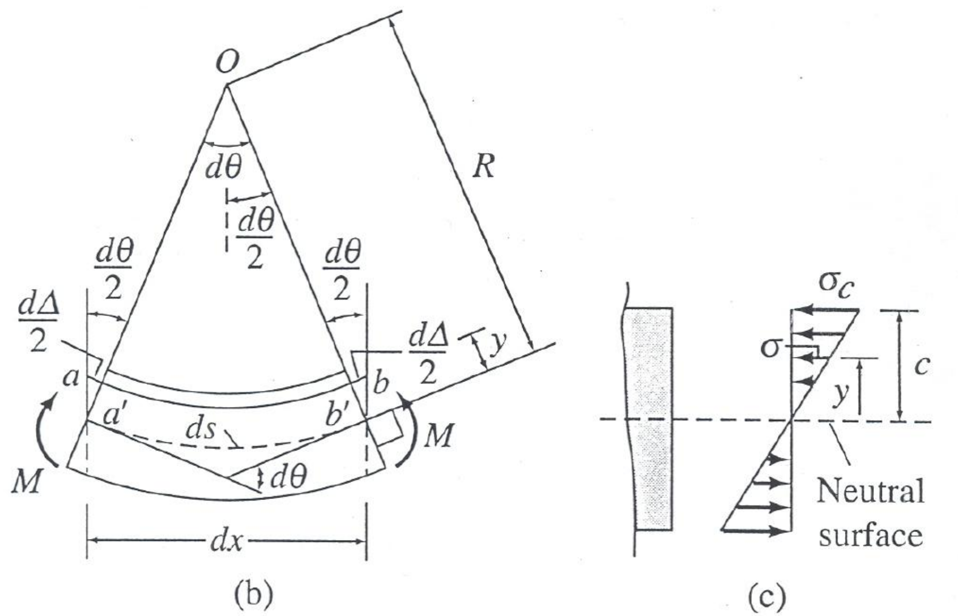


Bernoulli-Euler beam

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \qquad \frac{d\theta}{dx} = \frac{M}{EI}$$

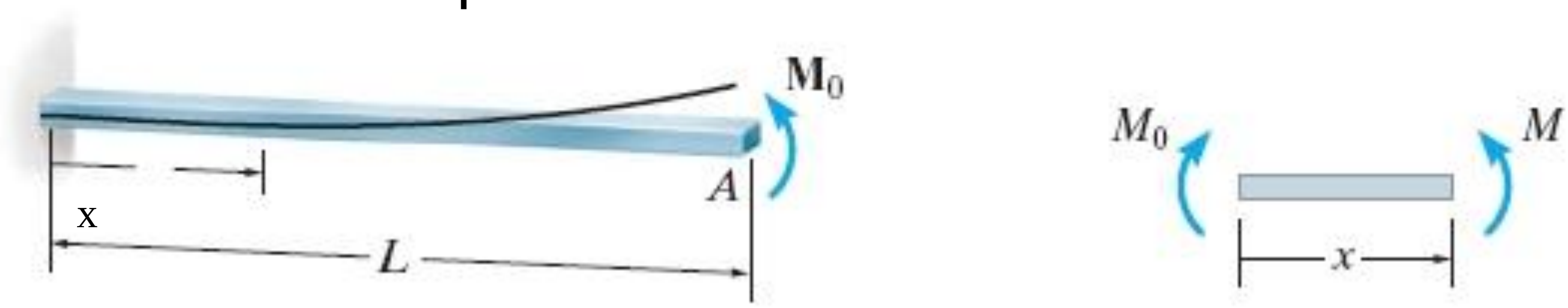
$$y = \iint \frac{M}{EI}\,dx\,dx \qquad \theta = \int \frac{M}{EI}\,dx$$

apply boundary conditions



Example 8.3

The cantilevered beam is subjected to a couple moment M_o at its end. Determine the eqn of the elastic curve. EI is constant.



(a) (b)

$$EI\frac{d^2v}{dx^2} = M_o$$

$$EI\frac{dv}{dx} = M_o x + C_1$$

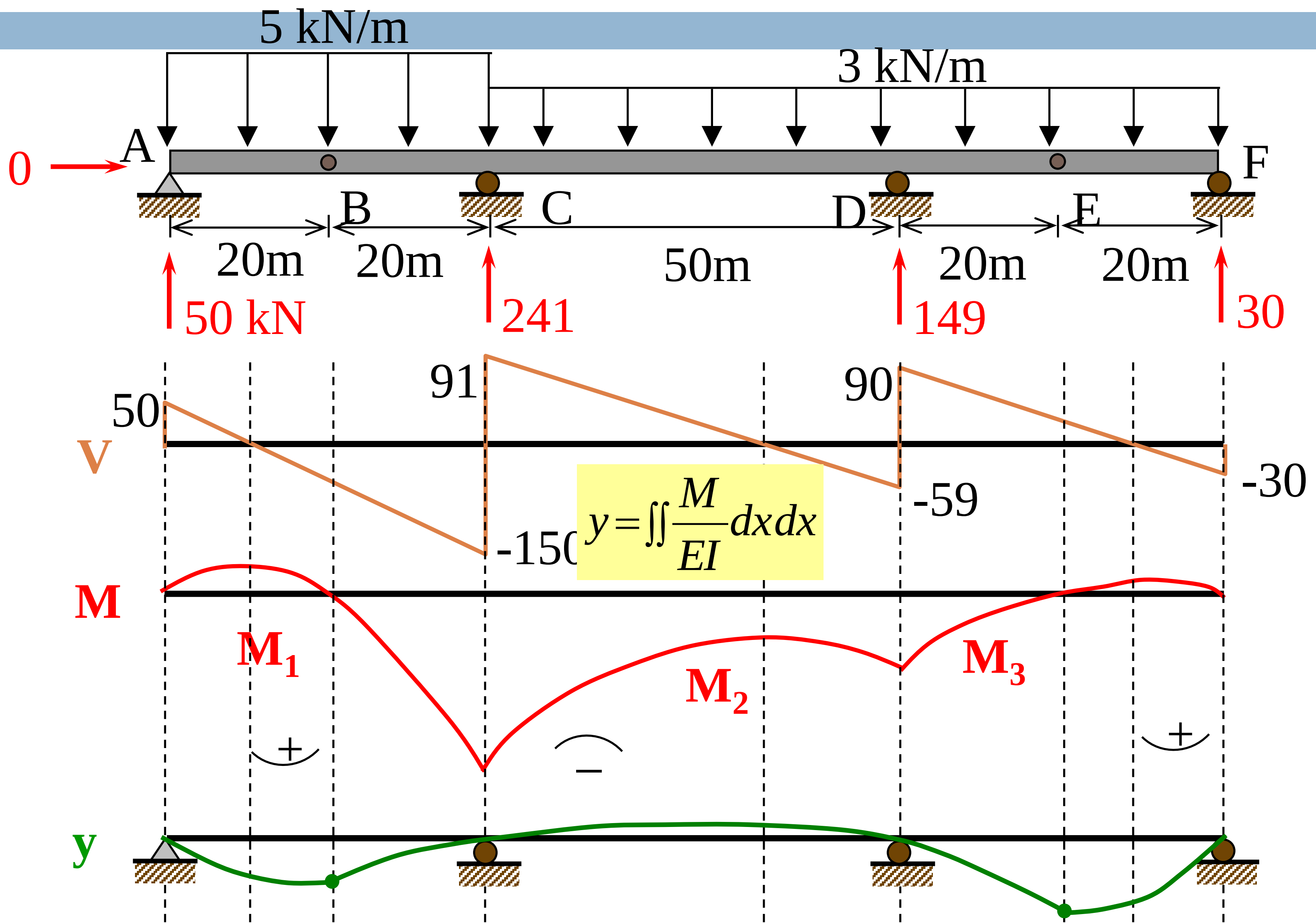
$$EI\,v = \frac{M_o x^2}{2} + C_1 x + C_2$$

at x = 0
dv/dx = 0 & v = 0
$C_1 = C_2 = 0$.

$$\theta = \frac{M_o x}{EI}; \quad v = \frac{M_o x^2}{2EI}$$

Max $$\theta_A = \frac{M_o L}{EI}; \quad v_A = \frac{M_o L^2}{2EI}$$

Direct Integration



$$y = \iint \frac{M}{EI} dx\,dx$$

Moment-Area Theorems

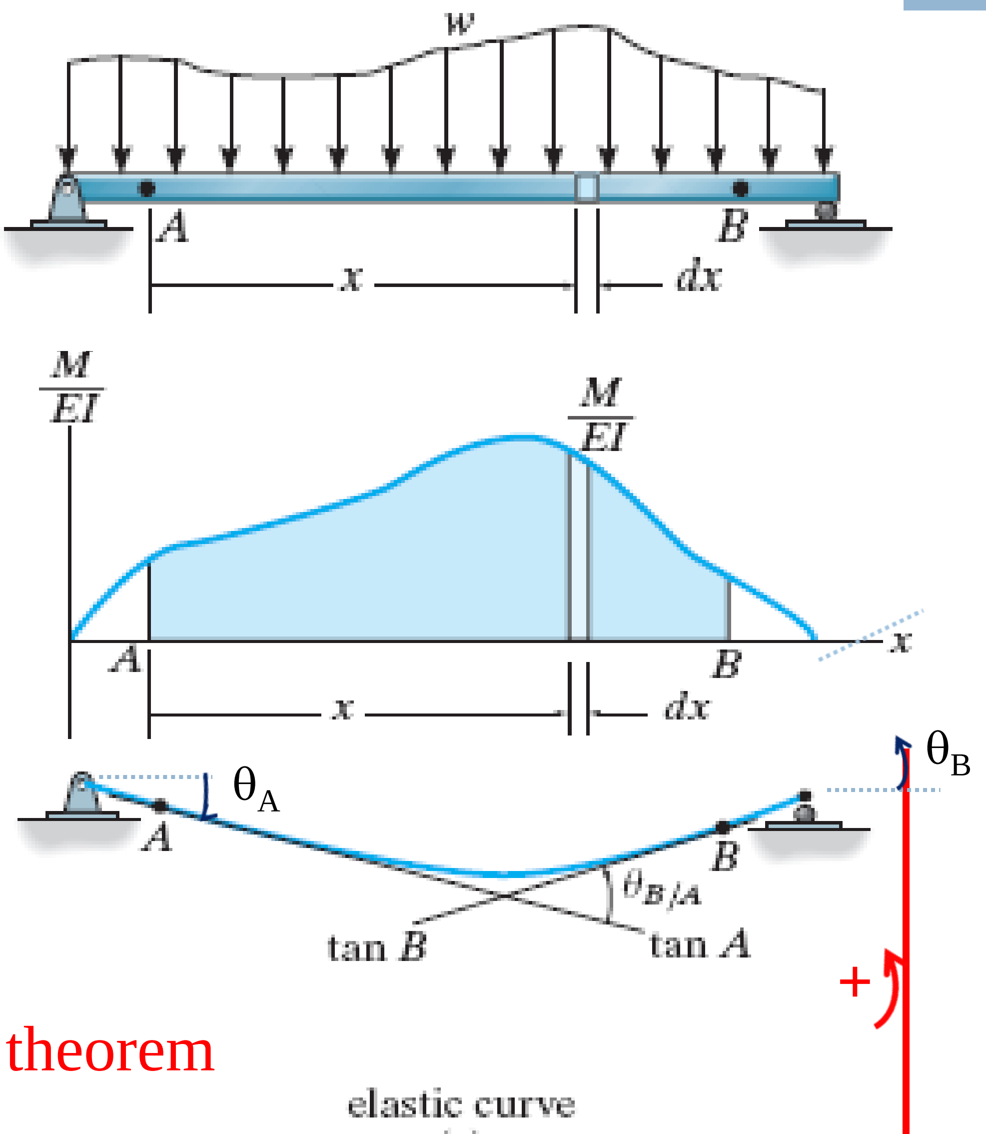
$$\frac{d^2y}{dx^2} = \frac{M}{EI} \qquad \frac{d\theta}{dx} = \frac{M}{EI}$$

$$d\theta = \left(\frac{M}{EI}\right)dx$$

$$\theta_B - \theta_A = \int_A^B \frac{M}{EI}\,dx$$

$\theta_{B/A}$

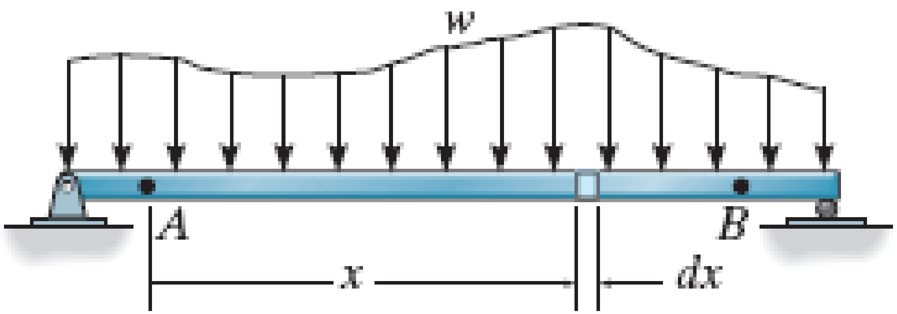
1st moment-area theorem

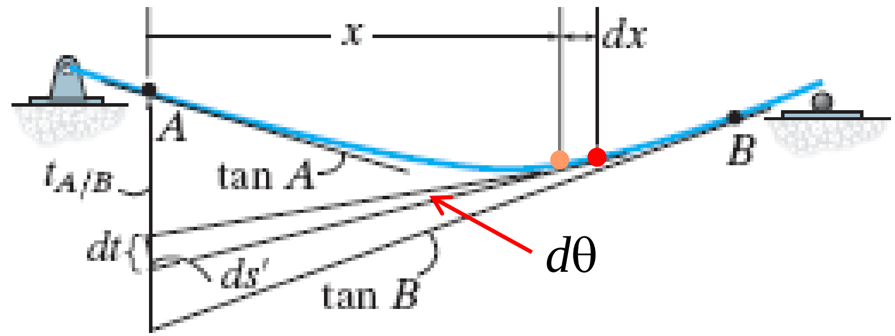


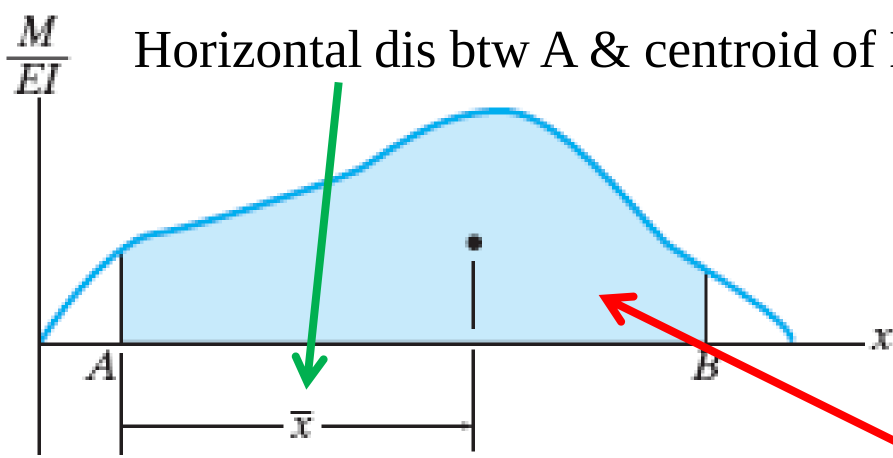
Moment-Area Theorems

- 2nd moment-area theorem

$$\frac{d\theta}{dx} = \frac{M}{EI}$$







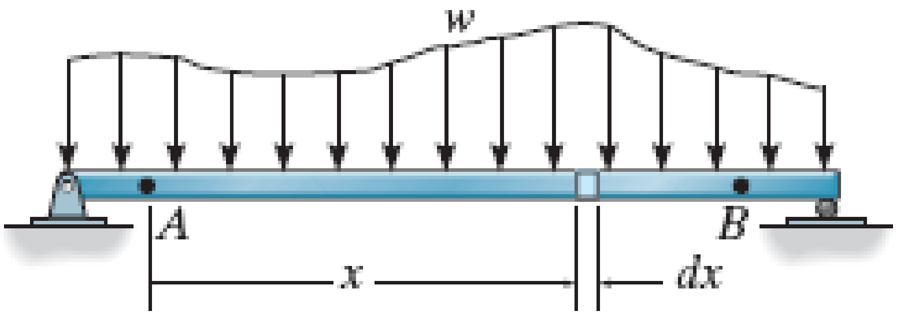
Horizontal dis btw A & centroid of $M/EI_{A\text{-}B}$

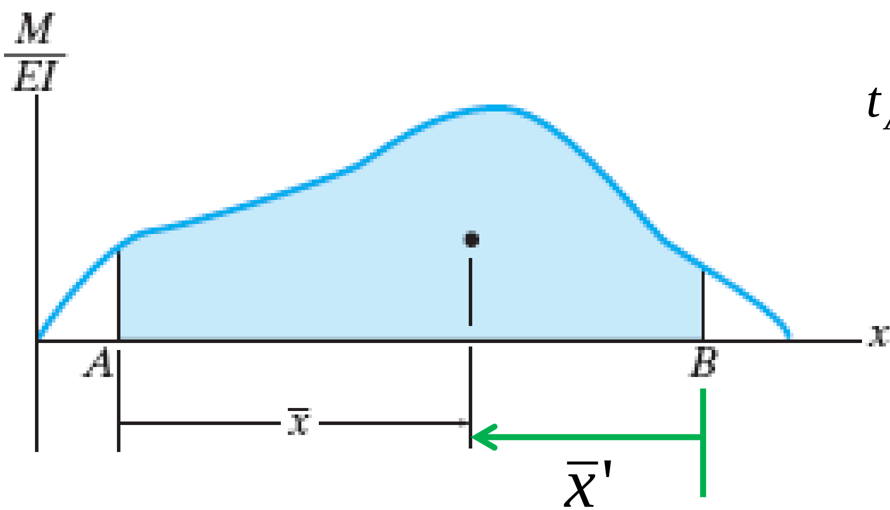
$$dt = x\,d\theta = x\frac{M}{EI}dx$$

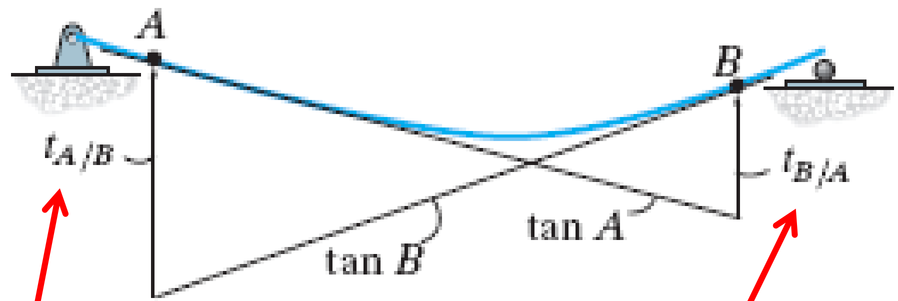
$$t_{A/B} = \int_A^B x\frac{M}{EI}dx = \bar{x}\int_A^B \frac{M}{EI}dx$$

M/EI area btw A & B

Moment-Area Theorems





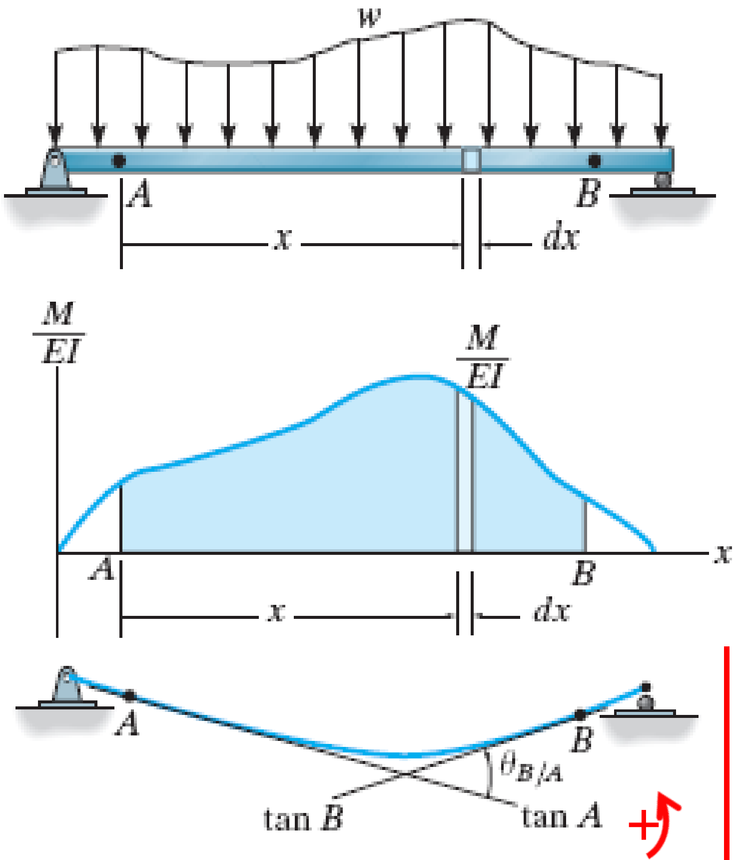


$$t_{A/B} = \int_A^B x \frac{M}{EI}\, dx = \bar{x} \int_A^B \frac{M}{EI}\, dx$$

$$t_{B/A} = \int_B^A x \frac{M}{EI}\, dx = \bar{x}' \int_B^A \frac{M}{EI}\, dx$$

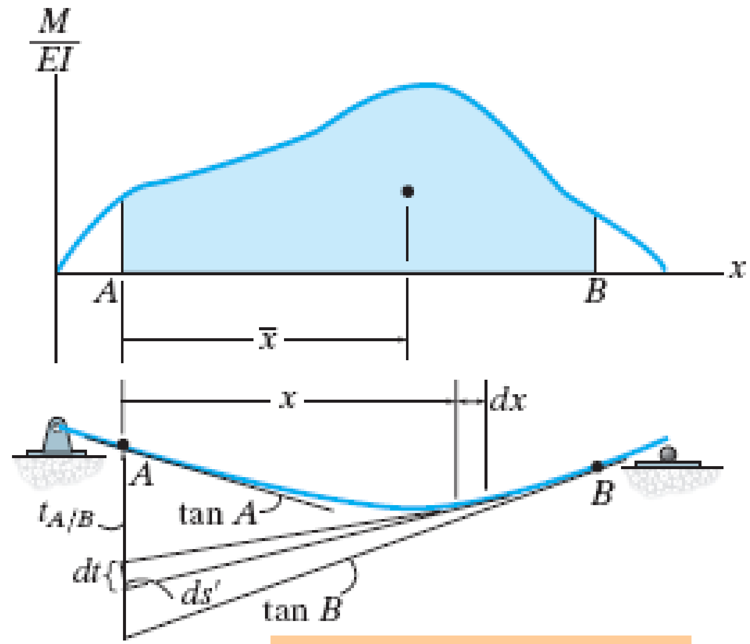
Moment-Area Theorems

1st moment-area theorem



$$\theta_{B/A} = \int_A^B \frac{M}{EI}\,dx$$

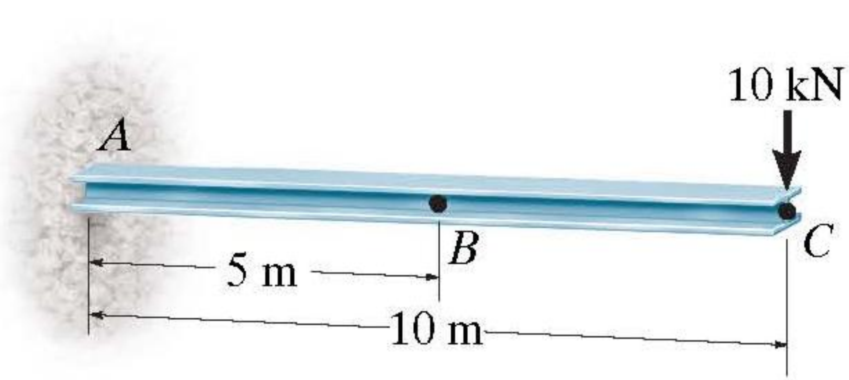
2nd moment-area theorem

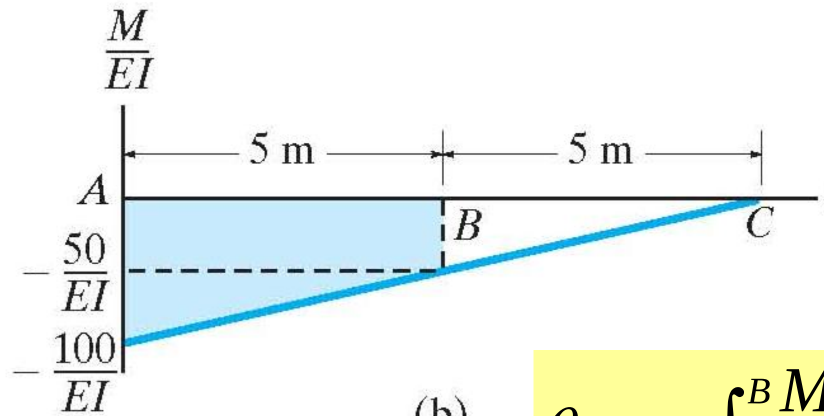


$$t_{A/B} = \bar{x}\int_A^B \frac{M}{EI}\,dx$$

Example 8.5

Determine the slope at points B & C of the beam. Take E = 200GPa, I =





(b)

$$\theta_{B/A} = \int_A^B \frac{M}{EI}\,dx$$

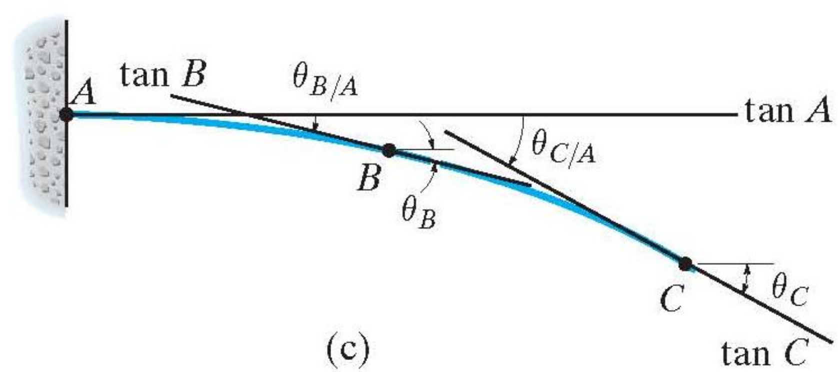
$$\theta_B = \theta_{B/A}; \quad \theta_C = \theta_{C/A}$$

$$\theta_B = \theta_{B/A}$$

$$= -\left(\frac{50kNm}{EI}\right)(5m) - \frac{1}{2}\left(\frac{100kNm}{EI} - \frac{50kNm}{EI}\right)(5m)$$

$$= -\frac{375kNm^2}{EI} = -0.00521\,rad$$

$$\theta_C = \theta_{C/A} = -0.00694 rad$$



(c)

Conjugate-Beam Method

- Mathematical analogy

$\frac{M}{EI}$ **-slope-deflection**	**Load-shear-moment**
$\frac{d\theta}{dx}=\frac{M}{EI}$	$\frac{dV}{dx}=w$
$\frac{dy}{dx}=\theta$	$\frac{dM}{dx}=V$
$\frac{d^2y}{dx^2}=\frac{M}{EI}$	$\frac{d^2M}{dx^2}=w$

Conjugate-Beam Method

- Mathematical equivalence

$\frac{M}{EI}$ -slope-deflection	Load-shear-moment
$\frac{M}{EI}$	w
θ	V
y	M

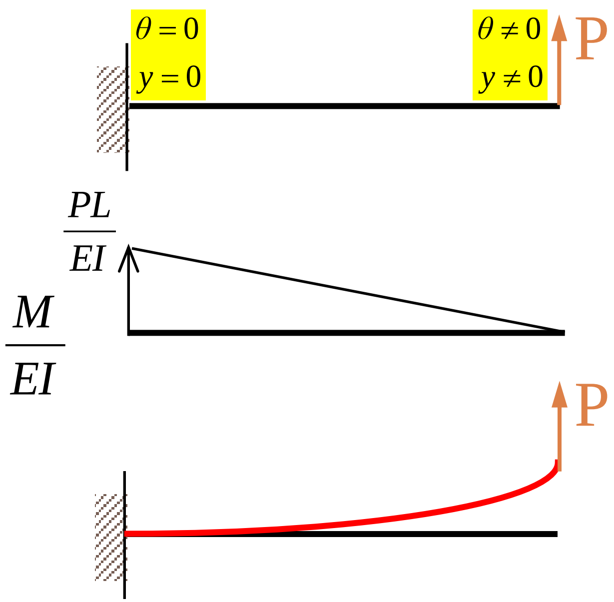
Actual beam — Conjugate beam

$\frac{M}{EI}$ -slope-deflection	Load-shear-moment
$\frac{d\theta}{dx} = \frac{M}{EI}$	$\frac{dV}{dx} = w$
$\frac{dy}{dx} = \theta$	$\frac{dM}{dx} = V$
$\frac{d^2y}{dx^2} = \frac{M}{EI}$	$\frac{d^2M}{dx^2} = w$

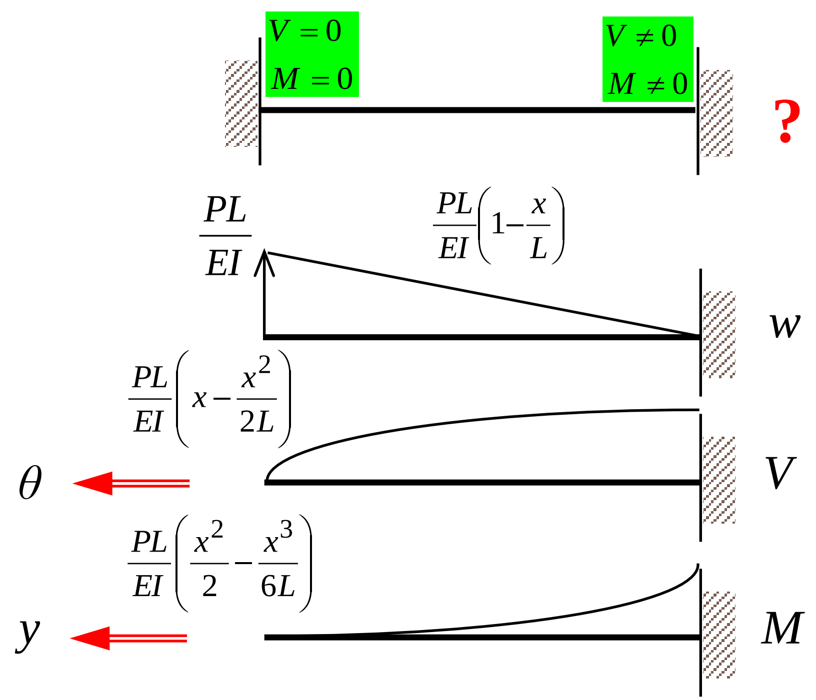
Conjugate-Beam Method

$\frac{M}{EI}$ -slope-deflection	Load-shear-moment
$\frac{M}{EI}$	w
θ	V
y	M
Actual beam	Conjugate beam

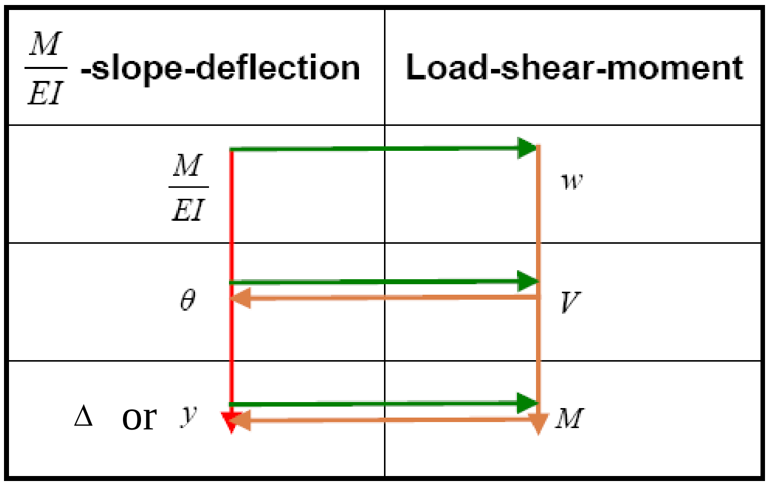
Actual beam



Conjugate beam



Conjugate-Beam Method

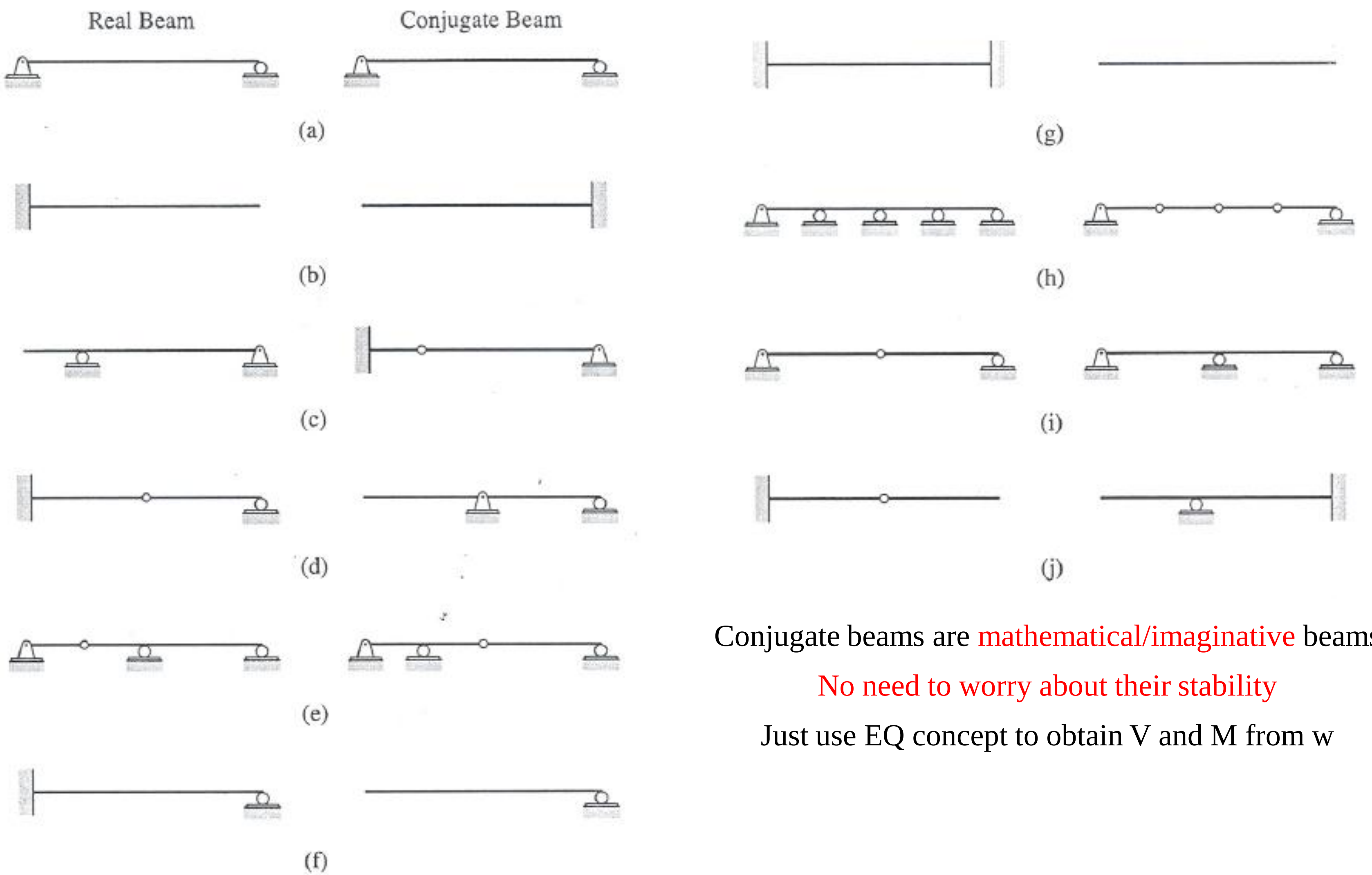


Actual beam   Conjugate beam

TABLE 8–2

	Real Beam		Conjugate Beam	
1)	θ $\Delta = 0$	pin	V $M = 0$	pin
2)	θ $\Delta = 0$	roller	V $M = 0$	roller
3)	$\theta = 0$ $\Delta = 0$	fixed	$V = 0$ $M = 0$	free
4)	θ Δ	free	V M	fixed
5)	θ $\Delta = 0$	internal pin	V $M = 0$	hinge
6)	θ $\Delta = 0$	internal roller	V $M = 0$	hinge
7)	θ Δ	hinge	V M	internal roller

Conjugate-Beam Method



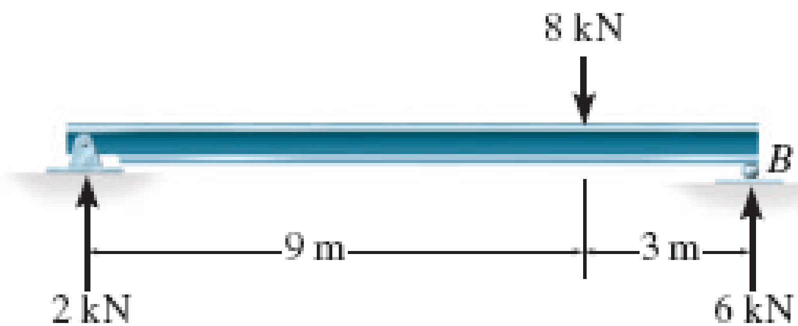
Conjugate beams are mathematical/imaginative beams

No need to worry about their stability

Just use EQ concept to obtain V and M from w

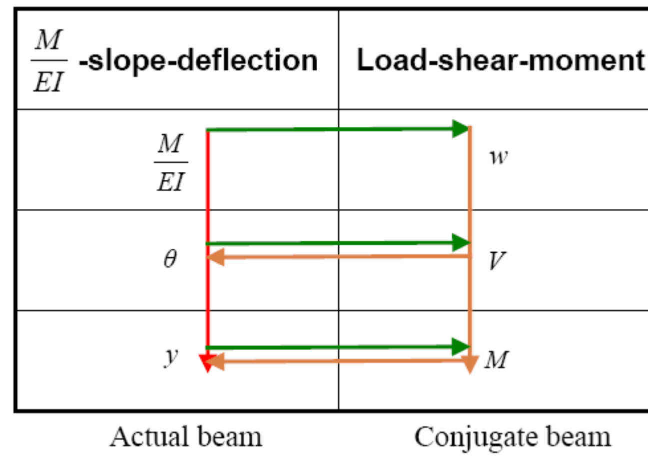
Example 8.5

Determine the max deflection of the steel beam. The reactions have been computed. Take E = 200GPa, I = 60(10^6)mm^4

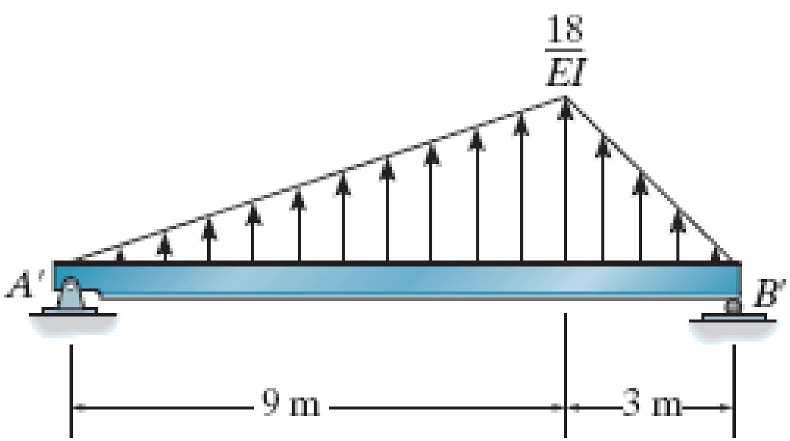


real beam

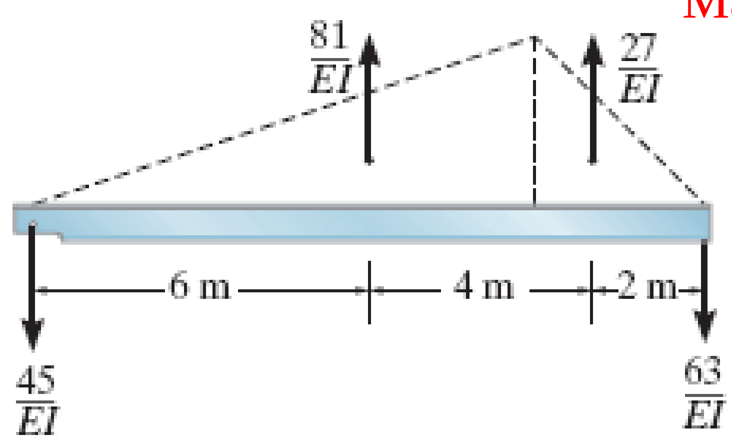
Max disp



Max M



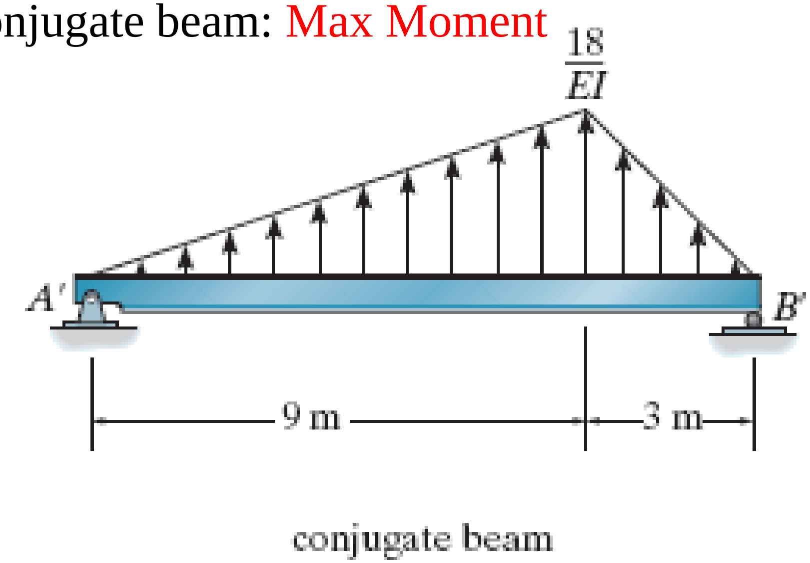
conjugate beam

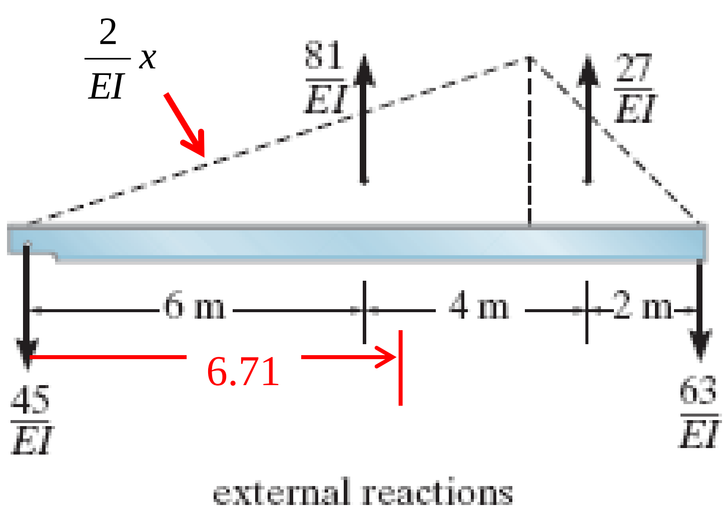


external reactions

Solution

Conjugate beam: Max Moment





Note: $\dfrac{dM}{dx} = V = 0$

when $M = M_{max}$

$$-\frac{45}{EI} + \frac{1}{2}\left(\frac{2x}{EI}\right)x = 0$$

$$x = 6.71m \quad (0 \le x \le 9m) \;\; OK$$

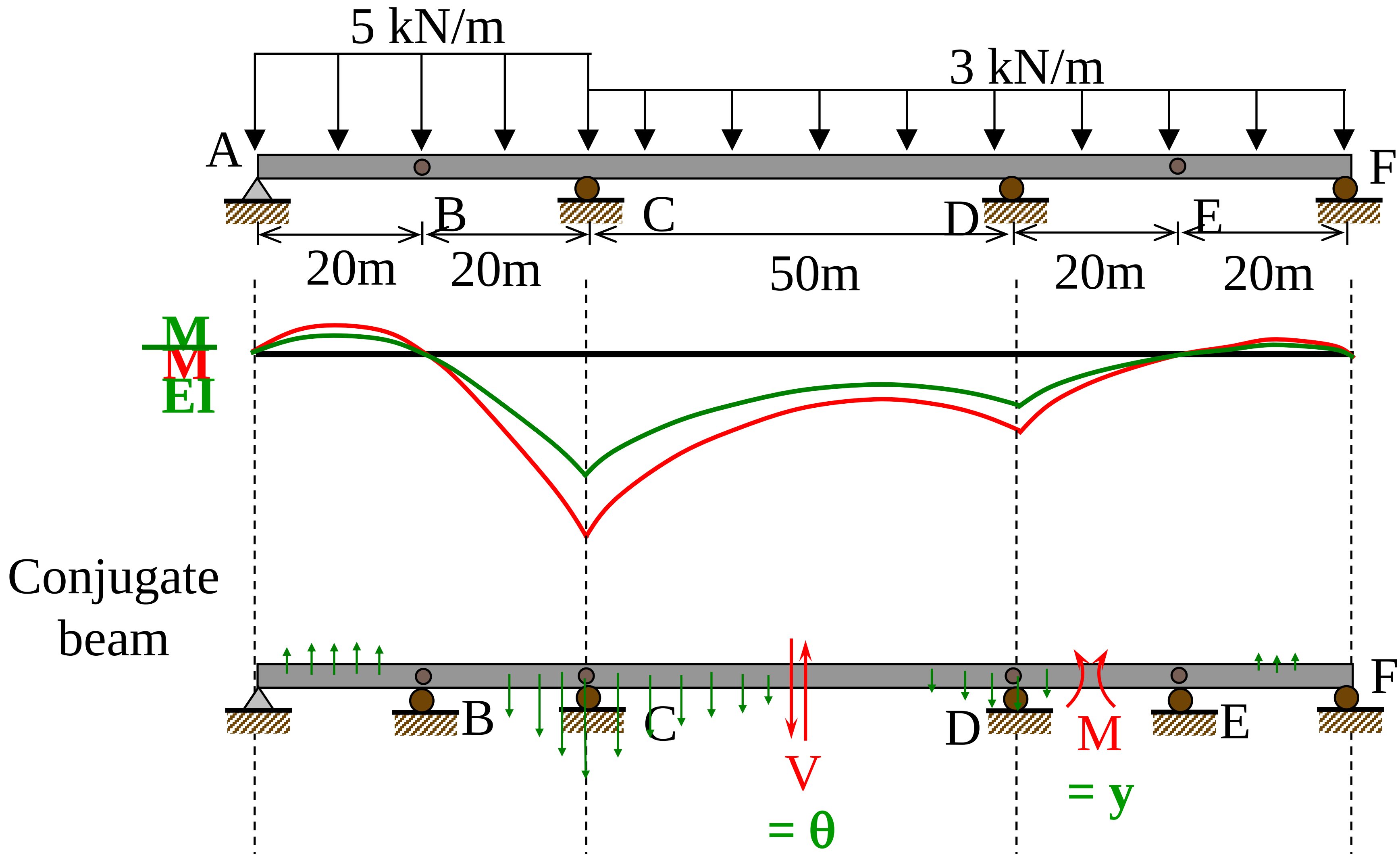
$$\frac{45}{EI}(6.71) - \left[\frac{1}{2}\left(\frac{2(6.71)}{EI}\right)6.71\right]\frac{1}{3}(6.71) + M' = 0$$

$$\Delta_{max} = M' = -\frac{201.2kNm^3}{EI}$$

$$= \frac{-201.2kNm^3}{[200(10^6)kN/m^2][60(10^6)mm^4(1m^4/(10^3)^4 mm^4)]}$$

$$= -0.0168m = -16.8mm$$

Conjugate-Beam



Summary

- Can you plot a qualitative deflection shape for a structure under loads (need to show 1^{st} & 2^{nd} order information)?
- Can you calculate structural deformation (displacement & rotation) using
  - Direct integration?
  - Moment-area principle?
  - Conjugate-beam method?